Evaluating Technology for Teaching Mathematics

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Use a framework to evaluate a tool: Is it mathematically sound? Does it offer opportunities for student engagement with little distraction? Will it afford students the chance to develop their own ideas?

Such digital technologies as calculators, handheld devices, computer software, Internet-based applets, and mobile applications can support students as they investigate mathematical ideas, develop mathematical conjectures, visualize abstract mathematical concepts, and understand concepts (NCTM 2000). In addition, technology can help students make sense of representations of mathematical ideas and make connections among the representations by providing a variety of algebraic, graphical, or geometric representations of mathematical concepts (NCTM 2014). However, the extent to which technological tools can do so depends on the selection of the tool and its implementation in the classroom. In other words, "the effective use of technology in the mathematics classroom depends on the teacher" (NCTM 2000, p. 25). But how, as a teacher, do you know whether a technological tool will be effective? In this article, we provide a framework that teachers can use to evaluate technological tools. We evaluate two examples that are designed to teach the same mathematical content, and we discuss the reasons for selecting one tool over the other, based primarily on the learning goal. When using this framework, we encourage teachers to align their evaluations with clear learning goals because effective teaching of mathematics requires teachers to set specific learning goals of a lesson and use the goals to guide their instructional decisions during planning and teaching the lesson (NCTM 2014).

A FRAMEWORK FOR EVALUATING A TECHNOLOGICAL TOOL

The development of the framework and guiding questions was based, in part, on our study (Smith, Shin, and Kim 2017), in which we examined how prospective and practicing

Table 1 This framework can be used to evaluate technological toos for teaching and learning mathematics.

Fidelity	Descriptions (Dick 2008)	Questions to Consider When Evaluating and Selecting Technological Tools
Pedagogical fidelity	How well the technological tool allows students to “do” mathematics without difficulty and to manipulate and not be distracted or limited by technical features	Is the tool difficult to use? Does the tool include clear instructions and directions on how to use it? Are there features that distract students from learning? How well does the tool allow students to interact with the mathematical object (e.g., shape, figure, table, plot, formula, equation) and take mathematical actions? How well does the tool offer students the opportunity to explore and develop conjectures and generalizations? How accessible is the tool for all students and does the tool offer customization or accommodations?
Mathematical fidelity	How well a mathematical object in the technological tool represents the underlying mathematical properties of the object with mathematical accuracy	How accurately does the tool represent the mathematics? Does the tool display mathematical formulas correctly, including basic assumptions? (Adapted from Bokhove and Drijvers 2010) What mathematical misconceptions may students develop while using the tool?
Cognitive fidelity	How well the technological tool reflects students’ cognitive actions with emphasis on illuminating mathematical thinking processes rather than simply arriving at the final results	How well does the tool show the ways in which the solution is produced? Does the tool simply display the final results? How well does the tool’s solution method resemble your students’ methods? Does the tool allow multiple solution methods? How well does the tool allow you to gain insight into how students are thinking?

mathematics teachers evaluate online applets designed for teaching and learning the Triangle Inequality theorem. To develop the framework, we also used the work of Dick (2008), who argued that technological tools designed for teaching and learning mathematics should have pedagogical, mathematical, and cognitive fidelity. Bos (2009) stated that a technological tool with a high degree of pedagogical, mathematical, and cognitive fidelity positively affects students’ mathematical achievement. For each type of fidelity in the framework, we provide questions that teachers should consider when evaluating and selecting tools and that are based on the NCTM technology standard (2000, 2014), the work of Bokhove and Drijvers (2010), and our own research (Smith, Shin, and Kim 2017) (see **table 1**).

1. *Is the technological tool pedagogically sound?*

Mathematics teachers must consider whether a technological tool allows students to pay attention to mathematics with as few distractions as possible. If the tool is difficult to use or does not provide directions, students and teachers may get frustrated as they determine how to use it. Even if the tool furnishes some instructions, if the instructions are not clear or do not make sense to both teacher and students, the mathematics lesson will likely turn into a computer class because the teacher and students will spend too much time trying to understand how to use the tool.

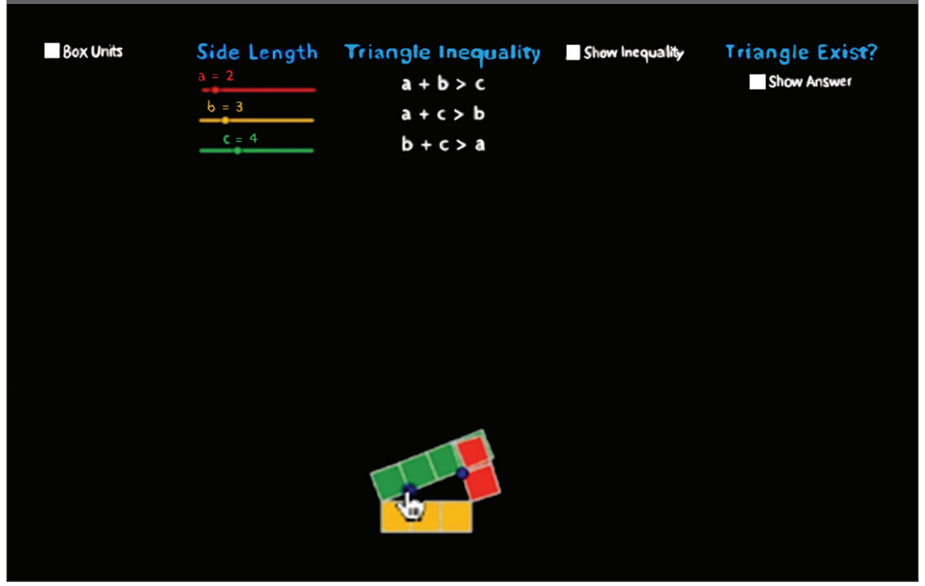
Mathematics teachers also must consider whether a tool includes features that distract students from learning mathematics. The excessive use of colors to represent mathematical objects and the inclusion of unnecessary objects are two examples of features that may distract students from understanding the desired mathematical concepts.

While evaluating online applets designed to explore the Triangle Inequality theorem, Abby, a teacher in our study, said, “In terms of the mathematics, I think that in this case, with the angle measures, since we’re not really concerned with angle measures, it could potentially distract.” Although angle itself is an important concept in geometry, Abby recognized that to develop the Triangle Inequality theorem, her students’ attention should focus on the three side lengths of a triangle.

Considering how a technological tool allows students to interact with objects within the tool is also important for mathematics teachers, rather than just using the tool to present mathematical concepts or theories (NCTM 2014). For example, teachers should consider whether a tool includes such features as dynamic motion (i.e., the ability to move an object, preserving the underlying mathematical properties of the object) and linked representation (e.g., a graph of an equation changes when the associated table is changed), animations, and the types of feedback that students receive after completing work.

To engage students in doing mathematics, teachers need to examine how a technological tool presents students with opportunities to develop and test conjectures that ultimately lead to generalizations. For example, teachers could examine whether a tool affords students the opportunity to create a variety of examples or representations, make conjectures, or observe the consequences of their actions within the tool.

Fig. 1 This technological tool represents segments as a sum of squares.

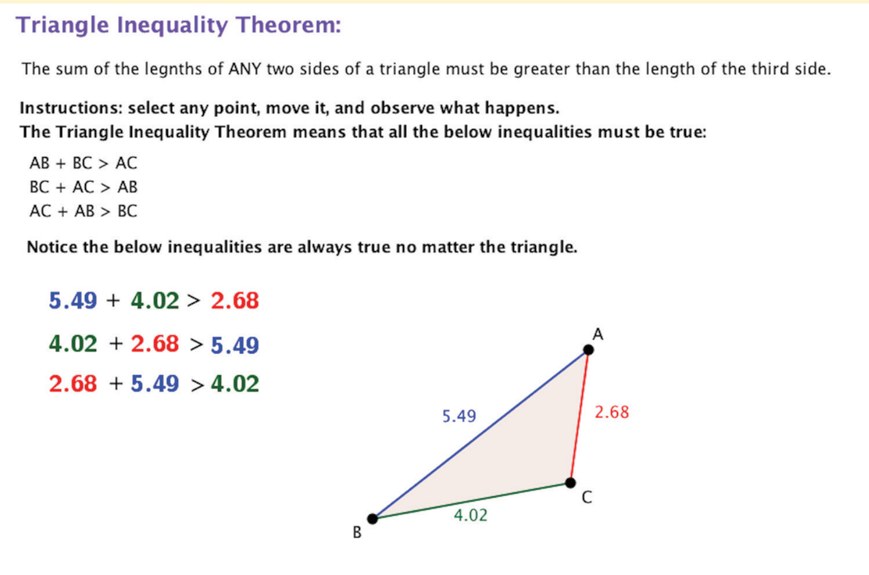


All students should have the opportunity to engage in meaningful mathematics learning, and teachers should consider whether a tool can accommodate all their students’ learning needs or can offer customization. For example, a tool that allows English language learners to toggle between English and the students’ native language could allow students to develop an understanding of the mathematics while also developing their English language skills. Students with vision difficulties may need zoom features, so that they can better see and interact with the mathematical objects, and read-aloud features, so that they can hear the instructions and questions.

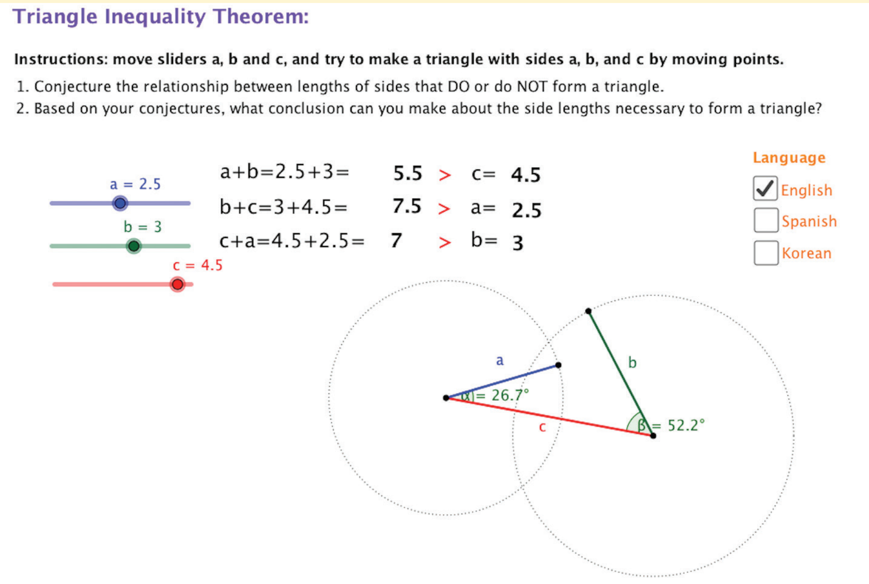
2. *Is the technological tool mathematically accurate?*

Teachers should evaluate the mathematical accuracy of the tool and its representations. At times, designers may place a greater priority on ease of use rather than mathematical faithfulness (Dick 2008). If the tool is not mathematically accurate, students may have difficulty developing the appropriate understanding of the concept. For example, one applet that mathematics teachers in our study evaluated portrayed sides of a triangle as a length of unit squares (see **fig. 1**). A middle school teacher who had issues with this representation said, “We just found it confusing. . . . Do you match up the boxes, or do you match up

Fig. 2 Applet 1 and applet 2 contain various information about the Triangle Inequality theorem.



(a) Applet 1



(b) Applet 2

their vertices? Where do you match them up to create the triangle?" To represent segments as rectangles or a length of unit squares may help some students clearly see the lengths of the segments by counting the number of squares. However, it may confuse students as they drag and explore whether the given three lengths of sides construct a triangle.

Sometimes a tool displays mathematical formulas (see **fig. 1**). Whether formulas are correctly displayed and include any underlying assumptions are important considerations for mathematics teachers. For example, an online applet designed to explore the Triangle Inequality theorem may display the inequality $a + b > c$. Although this seems to be correct, the theorem states that the sum of the lengths of *any* two sides of a triangle must be greater than the length of the third side. That is, the three lengths of sides $a = 3$, $b = 4$, $c = 1$ satisfy the inequality $a + b > c$, but they do not form a triangle. Without careful use of this applet by teachers, it may cause students to have misconceptions while exploring the theorem. For the formula to be correct, the two additional inequality statements ($a + c > b$, $b + c > a$) or the assumption $a \le b \le c$ should be included (see **fig. 1**).

3. *Is the technological tool cognitively reflective?*

Sometimes, mathematics teachers need to consider how well a technological tool shows the process being used by the tool itself to develop the final answer or result. For example, teachers may easily find many online resources to help students solve linear systems of equations. One such tool may ask students to enter both linear equations, but the output is only an answer (e.g., $x = 3$, no solution, or infinite solutions). Although students can find the answers quickly and accurately using this tool, they are unlikely to develop a conceptual understanding or procedural fluency. Teachers should consider whether and how well the tool displays the algebraic process to solve the linear systems of equations and further connects the algebraic process to a geometric representation.

In addition, teachers need to examine how well a tool reflects students' possible actions and choices while using it. Although a technological tool may use a correct process to solve a problem or derive a mathematical concept, it may not be the process that students would typically use. By skipping several steps, a tool may use techniques that are more sophisticated than those with which

Table 2 Summary of evaluation of two applets using the framework

Fidelity	Questions	Applet 1	Applet 2
Pedagogical fidelity	Difficult to use? Includes clear instructions and directions?	Not difficult to use Includes clear instructions	Not difficult to use Includes clear instructions
	Distracting features for learning?	None	Two interior angles and their measures
	Interacting with the mathematical objects?	Provides opportunities to change the lengths of segments and drag each point	Provides opportunities to change the lengths of segments and drag each point
	Offering opportunities to develop conjectures and generalizations?	Does not provide any opportunity to identify cases that do not form a triangle	Provides opportunities to create conjectures, test them, and generalize them, allowing students to see both cases that do and do not form a triangle
	How accessible is the tool for all students, and does the tool offer customization or accommodations?	Allows students to zoom	Allows students to zoom and select the instruction's language (English, Spanish, or Korean)
Mathematical fidelity	Mathematically accurate?	Segment lengths range from 0 to infinite with 0.01 increments	Segment lengths range from 0 to 5 with 0.5 increments
	Displaying mathematical formulas correctly?	Always presents the inequality sign even when the relationship is equal	Displays correctly
	Causing mathematical misconceptions?	Possibly because the inequality statement is not always true	None
Cognitive fidelity	Showing the ways in which the solution is produced or displaying only the final results?	Displays the final result (i.e., theorem) as part of the instructions	Shows the ways in which the solution is produced visually and numerically
	Reflecting students' methods?	Does not reflects students' method because the triangle is already formed	Reflects students' method because it allows students to create the triangle themselves
	Allowing teachers to gain insight into how students are thinking?	None	Possibly based on the students' selection of segment lengths and interaction with the segments

students would be familiar (Dick 2008). For example, many graphing calculators use Newton's method to find zeros of a quadratic function by using successive tangent line approximations. Although this sophisticated method is valid, it is neither the method we typically teach students nor the method students would use on their own. Instead, students will use other methods, such as factoring, completing the square, or the quadratic formula, when finding zeros of a quadratic function. Teachers need to recognize that even though a process provided by a tool is valid, it is sometimes not the one we want students to learn and use.

USE OF THE FRAMEWORK

In this section, we illustrate how we used the framework to evaluate two online applets (see **figs. 2a–2b**) designed to teach the Triangle Inequality theorem to middle school students. A critical first step in evaluating and selecting a technological tool is to determine the learning goal. We wanted our students to predict, conjecture, and test conditions needed to form a triangle and ultimately develop the Triangle Inequality theorem. We then used the framework to evaluate each applet (see **table 2**) and selected one on the basis of our learning goal.

In terms of pedagogical fidelity, both applets contain clear instructions and do not seem to be difficult to use. Both applets have a zoom feature, and applet 2 allows students to select the language for the instructions. In addition, both applets allow students to have opportunities to interact with the mathematical objects by changing the lengths of segments and dragging each point. However, applet 2 includes two interior angles, named α and β, and their measures. These angle features may distract students from focusing on the side lengths. Using applet 2, students are more likely to develop conjectures and generalizations than when using applet 1 for two reasons. First, applet 1 states the Triangle Inequality theorem, and it gives students the opportunity only to verify the theorem. Applet 2 presents opportunities for students to create their own conjectures, test those conjectures by interacting with the applet, and generalize these conjectures. Second, applet 1 does not offer any opportunity for students to identify cases that do not form a triangle except for one case in which the sum of two sides equals the third side, whereas applet 2 allows students to interact with all cases.

With one major exception, both applets seem to be mathematically sound. However, applet 1 always presents the inequality sign, even when the relationship is equal (e.g., when $a = 1.5$, $b = 1.9$, and $c = 3.4$). In terms of segment lengths, applet 1's lengths have no upper bound and can be incremented by 0.01, whereas applet 2 provides sides whose lengths range from 0 to 5 with 0.5 increments. Thus, applet 2 is restricted to a finite number of possible examples. But because applet 1 is not always mathematically accurate, students could develop misconceptions (see **figs. 2a–2b**).

We also evaluated the two applets in terms of whether they were cognitively faithful. Applet 1 displays the Triangle Inequality theorem, whereas applet 2 provides students opportunities to explore and develop the theorem. In addition, applet 2's method is more likely to resemble the method that students would typically use to see if the segments form a triangle because the students can drag the segments to form the triangle themselves rather than the tool creating the triangle for them. Because applet 2 allows students to set the lengths of the segments and gives students the opportunity to try to create the triangle themselves, teachers could possibly gain insight into students' thinking and reasoning.

OLIVIER LE MOAL/THINKSTOCK

Although one applet may appear to be better than the other, your evaluation should consider your learning goals. Based on our learning goal, we believe applet 2, even with its flaws, is more appropriate for us than applet 1 because our learning goal was to allow students to explore and develop the theorem.

DISCUSSION

We believe the framework can be a powerful tool for mathematics teachers, particularly those with little experience with evaluation of technological tools. However, we recognize that the framework focuses heavily on mathematics and may not be very useful for evaluating general instructive tools, such as interactive white boards or clickers. In addition, when evaluating and selecting a tool, teachers need to consider how their students learn best and the classroom context because the features that may distract students from learning in one situation could be the same ones that enhance students' learning in other situations.

Furthermore, finding a technological tool with a high degree of pedagogical, mathematical, and cognitive fidelity is not easy. To ensure that a mathematics lesson with a tool will be effective, teachers must have clear learning goals for what they want students to learn from the lesson. For example, if the learning goal is to verify the Triangle Inequality theorem, a high degree of cognitive fidelity may be unnecessary because the goal focuses on the solution and not the method. Thus, applet 1 might be a better choice because it allows students to select any set of side lengths and see if the theorem is true. On the other hand, if the goal is to develop the theorem itself, teachers need to carefully select the technological tool that provides students the opportunity to interact with a mathematical object, and develop and test their own conjectures.

"Technology does not replace the mathematics teacher. . . . The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways" (NCTM 2000, p. 26). Thus, the evaluation and selection of which technological tools to use is extremely important. We believe that teachers who use this framework with clear learning goals in mind will select the technological tool that has the most potential to effectively assist students in meeting the learning goals.

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Ed. note: For a related article this month in *Mathematics Teacher*, see "Teaching with (Not Near) Technology," by Lindsay Reiten.

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